

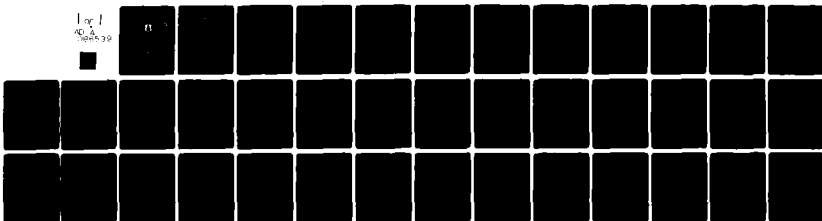
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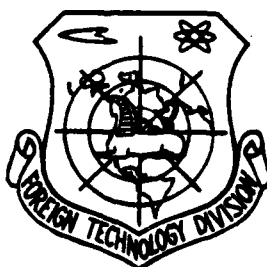
FOREIGN TECHNOLOGY DIVISION



EQUATIONS OF THE PROBLEM OF DETERMINING THE  
LOCATION OF A MOVING OBJECT BY GYROSCOPES  
AND ACCELEROMETERS

by

A. Yu. Ishlinskiy



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# EDITED TRANSLATION

(14)

FTD-ID(RS)T-0925-80

13 June 1980

(11) 13 Jun 80

MICROFICHE NR: FTD-80-C-000699

(6)

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English pages: 37

(21)

Edited trans. of

Source: Prikladnaya Matematika i Mekhanika (USSR)  
v. 21, No. 6, 1977, by [redacted]  
Vol. 21, Nr. 6, [redacted]

Country of origin: USSR

This document was translated by Carol S. Nack.

Requester: FTD/SDDS

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PREPARED BY:

TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WP.AFB, OHIO.

FTD-ID(RS)T-0925-80

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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b><i>А а</i></b>	A, a	Р р	<b><i>Р р</i></b>	R, r
Б б	<b><i>Б б</i></b>	B, b	С с	<b><i>С с</i></b>	S, s
В в	<b><i>В в</i></b>	V, v	Т т	<b><i>Т т</i></b>	T, t
Г г	<b><i>Г г</i></b>	G, g	У у	<b><i>У у</i></b>	U, u
Д д	<b><i>Д д</i></b>	D, d	Ф ф	<b><i>Ф ф</i></b>	F, f
Е е	<b><i>Е е</i></b>	Ye, ye; E, e*	Х х	<b><i>Х х</i></b>	Kh, kh
Ж ж	<b><i>Ж ж</i></b>	Zh, zh	Ц ц	<b><i>Ц ц</i></b>	Ts, ts
З з	<b><i>З з</i></b>	Z, z	Ч ч	<b><i>Ч ч</i></b>	Ch, ch
И и	<b><i>И и</i></b>	I, i	Ш ш	<b><i>Ш ш</i></b>	Sh, sh
Й й	<b><i>Й й</i></b>	Y, y	Щ щ	<b><i>Щ щ</i></b>	Shch, shch
К к	<b><i>К к</i></b>	K, k	Ъ ъ	<b><i>Ъ ъ</i></b>	"
Л л	<b><i>Л л</i></b>	L, l	Ы ы	<b><i>Ы ы</i></b>	Y, y
М м	<b><i>М м</i></b>	M, m	Ь ь	<b><i>Ь ь</i></b>	'
Н н	<b><i>Н н</i></b>	N, n	Э э	<b><i>Э э</i></b>	E, e
О о	<b><i>О о</i></b>	O, o	Ю ю	<b><i>Ю ю</i></b>	Yu, yu
П п	<b><i>П п</i></b>	P, p	Я я	<b><i>Я я</i></b>	Ya, ya

\*ye initially, after vowels, and after Ъ, ь; e elsewhere.  
When written as ě in Russian, transliterate as yě or ě.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh
cos	cos	ch	cosh	arc ch	cosh
tg	tan	th	tanh	arc th	tanh
ctg	cot	cth	coth	arc cth	coth
sec	sec	sch	sech	arc sch	sech
cosec	csc	csch	csch	arc csch	csch

Russian      English

rot      curl  
lg      log

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EQUATIONS OF THE PROBLEM OF DETERMINING THE LOCATION OF A MOVING  
OBJECT BY GYROSCOPES AND ACCELEROMETERS

A. Yu. Ishlinskiy (Moscow)

Summary

The problem of the so-called autonomous, i.e., without using external references, determination of the location of a moving object is of great practical significance. Until recently, this problem had virtually no chances for practical resolution because of the lack of precision of the sensors available to the engineer for this purpose, namely - newtonometers<sup>1</sup>, or accelerometers, gyroscopes, and integrators.

Footnote: <sup>1</sup>We are proposing that the instruments previously known in technology as accelerometers, or acceleration meters, subsequently be

called newtonometers, since they measure the combined action of both the forces of inertia of translational motion and the forces of gravity on their sensor. It is the projection of the resultant combination of these forces on a certain direction bound to the instrument, which we will subsequently call the axis of sensitivity of the newtonometer, which is measured.

We will point out that the above forces of inertia of translational motion should be determined, of course, relative to a coordinate system which is strictly bound to the instrument itself. Coriolis forces of inertia usually do not affect the instrument reading. End footnote

However, recently the creation of new elements [1] has already made it possible to solve this problem with satisfactory precision, as long as the duration of movement of the object is short.

This report considers the theory of one possible version of the problem of the autonomous determination of the location of a moving object.

The important problem of estimating the precision of the determination of location, which is affected by the presence of the

so-called instrument errors of the newtonometers, gyroscopes and other components of the system, is beyond the scope of this report and may become the object of special studies. Therefore, in the future we will assume that the above components operate without errors. Accordingly, the electromechanical system which solves the problem will be considered with the assumption that all of its parameters precisely correspond to their theoretical values, and that there are no technological errors (e.g., lack of precision of assembly and free strokes in transmissions). Furthermore, the initial conditions of small oscillations of this system can be arbitrary.

1. First we will give the solution of the known problem of autonomous determination during the movement of an object over an arc of the great circle of a certain nonrotational sphere S, whose center coincides with the center of the Earth (Fig. 1). In the simplest case, this corresponds to movement at a constant altitude above the equator. Here the determination of the location of the object relative to the Earth itself is reduced to simply timing the movement.

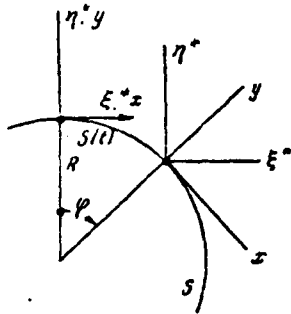


Fig. 1.

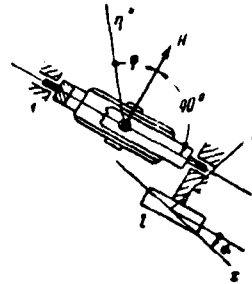


Fig. 2.

We will bind two systems of coordinates  $xy$  and  $\xi^*\eta^*$  having the same origin with a certain determined point of a moving object (Fig. 1). Henceforth, we will call this point the center of the object. The  $x$ -axis of coordinate system  $xy$  is directed along the velocity vector of the object  $v$ . Accordingly, the  $y$ -axis is the continuation of the Earth's radius passing through the center of the moving object. The direction of the axes of coordinate system  $\xi^*\eta^*$  is fixed (relative to stationary stars or, analogously, relative to sphere  $S$ ).

Suppose that the movements of both coordinate systems coincide at the initial point in time. Subsequently, coordinate system  $xy$  will be turned relative to progressively moving system  $\xi^*\eta^*$  by a certain



angle  $\phi$  which varies through time. This angle is related to the distance  $s = s(t)$  covered by the center of the object from the initial point by the relationship

$$\varphi = \frac{s}{R} \quad (1.1)$$

Here  $R$  is the radius of the arc of the great circle (the Earth's radius) over which the center of the object moves.

We will place a certain platform 1 stabilized by gyroscopes on the object (Fig. 2). In the simplest case, this platform is oriented by special servo systems so that it is always perpendicular to the vector of the intrinsic kinetic moment of a certain gyroscope (Fig. 2). The bearings of the axle of the outer ring of Cardan joints for the suspension of this gyroscope can be attached to a stabilizing platform.

Let the intrinsic kinetic moment of the gyroscope  $H$  lie in the plane  $\xi^*\eta^*$ , and a certain moment  $M = M(t)$  be applied to the axis of the outer ring of its suspension. In this case, the precession of the gyroscope axis occurs in the same plane  $\xi^*\eta^*$ . Following the gyroscope, the stabilized platform will rotate at an angular velocity

where  $\psi$  is the angle between the axis of natural rotation of the gyroscope and axis  $\eta^*$ .

Furthermore, (Fig. 2) we will place newtonometer 2 in the plane of the stabilized platform 1 and we will use  $a = a(t)$  to designate its current reading.

Let the axis of sensitivity of the newtonometer coincide with the x-axis. Then its reading  $a(t)$  (Fig. 3) is expressed by the formula

$$a(t) = \left(j - \frac{v^2}{R}\right) \sin \alpha + \frac{d^2 s}{dt^2} \cos \alpha \quad (1.3)$$

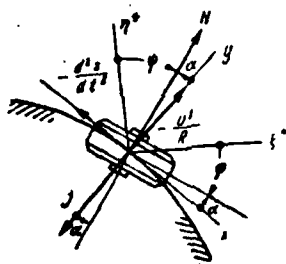


Fig. 3.

Here  $j$  is the gravitational force per unit mass, and the angle

$$\alpha = \varphi - \psi \quad (1.4)$$

is the deflection of the stabilized platform from horizontal direction (to be more precise, perpendicular to the Earth's radius). If the function  $a(t)$  obtained at the output of the newtonometer is integrated using special devices, as a result we can form the new function

$$K \int_0^t a(t) dt + m \quad (1.5)$$

where  $K$  and  $m$  are certain constant parameters. The necessary values of these parameters will be established later.

We can reproduce function (1.5) in the form of moment  $M(t)$  acting on the gyroscope of the stabilized platform. Then, plugging  $M(t)$  into formula (1.2) and integrating, we will have

$$\psi = \frac{K}{H} \iint_0^t a(t) dt^2 + \frac{m}{H} t + \psi_0 \quad (1.6)$$

where  $\psi_0$  is the angle of inclination of the stabilized platform to the horizon at the initial point in time  $t = 0$  (to be more precise, to the  $x$ -axis, or to axis  $\xi^*$ , which coincides with it at this instant).

Considering formulae (1.1) and (1.4) and relationship (1.6), we

will have the equation

$$\alpha = \frac{s}{R} - \frac{K}{H} \int_0^t a(t) dt - \frac{m}{H} t - \psi_0 \quad (1.7)$$

which makes it possible to set up the differential equation for the function  $\alpha = \alpha(t)$  and the initial conditions of this equation.

Actually, setting  $t = 0$  in equation (1.7) and noting that  $\alpha(0) = 0$ , we will have

$$\alpha(0) = -\psi_0 \quad (1.8)$$

which, of course, also follows directly from formula (1.4).

Subsequently, we will equate the derivatives of the left and right sides of equation (1.7). We will obtain

$$\frac{d\alpha}{dt} = \frac{1}{R} \frac{ds}{dt} - \frac{K}{H} \int_0^t a(t) dt - \frac{m}{H} \quad (1.9)$$

Whence it follows that the initial value of the derivative of function  $\alpha(t)$  through time is expressed by the formula

$$\frac{d\alpha(0)}{dt} = \frac{v(0)}{R} - \frac{m}{H} \quad \left( v(0) = \frac{ds(0)}{dt} \right) \quad (1.10)$$

Here  $v(0)$  is the initial value of the velocity of the center of the object relative to nonrotating sphere S.

Finally, if we differentiate both sides of equation (1.7) once

more and consider formula (1.3), we will arrive at the relationship

$$\frac{d^2\alpha}{dt^2} + \frac{K}{H} \left( j - \frac{v^2}{R} \right) \sin \alpha = \left( \frac{1}{R} - \frac{K}{H} \cos \alpha \right) \frac{ds}{dt} \quad (1.11)$$

With a given function  $s = s(t)$ , this relationship can be considered to be the second-order differential equation for the function  $\alpha = \alpha(t)$ , i.e., for the angle of inclination of the stabilized platform to the horizon. The initial conditions of this differential equation are determined by equations (1.8) and (1.10).

2. Differential equation (1.11) has a partial integral

$$\alpha \equiv 0 \quad (2.1)$$

which is extremely important for solving the problem of the autonomous determination of the position of a moving object, as long as the equation

$$\frac{1}{R} = \frac{K}{H} \quad (2.2)$$

is satisfied, as it is not hard to see, and the initial conditions are such that

$$\alpha(0) = 0, \quad \frac{d\alpha(0)}{dt} = 0 \quad (2.3)$$

Equation (2.2) determines the value of parameter  $K$ . Together with equation (1.8), the first condition of (2.3) leads to the requirement that there is no initial deflection of the stabilizing platform from the horizon, i.e., the equation

$$\psi_0 = 0 \quad (2.4)$$

is satisfied.

According to the second condition of (2.3) and formula (1.10), we will have

$$\frac{\omega(0)}{R} = \frac{m}{H} \quad (2.5)$$

This determines the value of parameter  $m$  in the device which forms function (1.5).

Thus, when conditions (2.4) and (2.5) are observed, as well as equation (2.2), the stabilized platform will remain horizontal (to be more precise, it will be perpendicular to the Earth's radius) with any law of the movement of the center of the object  $s = s(t)$ .

However, if  $\alpha \neq 0$ ,  $a(t) = d^2s/dt^2$  in accordance with formula (1.3) and, consequently, with consideration of conditions (2.5) and (2.2), function (1.5) will assume the form:

$$K \left[ \int_0^t \frac{d^2 s}{dt^2} dt + v(0) \right] = Kv(t) \quad (2.6)$$

Thus, in this case, with precision down to the constant factor  $K$ , function (1.5) is the current value of the velocity of the object  $v = v(t)$  relative to nonrotating sphere  $S$ . Therefore, if we now apply function (1.5) to the second integrating device, with the same assumptions, at its output we will obtain the unknown distance  $s(t)$  covered by the center of the object from the original position (also with precision down to the factor  $K$ ).

3. In the same general case, when  $\alpha \neq 0$ , the value

$$\int_0^t \left[ K \int_0^t a(t) dt + m \right] dt \quad (3.1)$$

generated by the above integrating devices differs from the product  $Ks(t)$ . According to equation (1.7) and condition (2.2), we will have

$$s(t) - \frac{1}{K} \int_0^t \left\{ \int_0^t Ka(t) dt + m \right\} dt = R\alpha(t) + R\psi_0 \quad (3.2)$$

Thus, the value of the error  $\Delta s$  in determining the distance by the above method is

$$\Delta s = R[\alpha(t) - \alpha(0)] \quad (3.3)$$

where  $\alpha(t)$  is determined by differential equation (1.11) with initial conditions (1.8) and (1.10) and the observation of equation (2.2).

The angle  $\alpha(t)$  of the inclination of the stabilized platform to the x-axis can be considered to be extremely small. Therefore, dropping the terms with the second order of smallness relative to  $\alpha$  from equation (1.11) and considering equation (2.2), we will arrive at the following homogeneous linear differential equation:

$$\frac{d^2\alpha}{dt^2} + \frac{1}{R} \left( j - \frac{v^2}{R} \right) \alpha = 0 \quad (3.4)$$

With the known approximation we can assume that

$$j - \frac{v^2}{R} = g = \text{const} \quad (3.5)$$

where  $g$  is the value of the acceleration of the force of gravity in the region of movement of the object. In this case, the solution to equation (3.4) is in the form of a geometric function:

$$\alpha(t) = \alpha(0) \cos vt + \frac{1}{v} \frac{d\alpha(0)}{dt} \sin vt \quad \left( v^2 = \frac{g}{R} \right) \quad (3.6)$$

whose period

$$T = 2\pi \sqrt{R/g} \approx 84.4 \text{ min.}$$



is known in gyroscope theory as the Schuler period.

Thus, in general, according to formulae (3.3) and (3.6), the error in the autonomous determination of the location of the object by the above method fluctuates.

4. We can also suggest other methods of determining the location of an object which generally lead to the same results.

For example, suppose the platform is stabilized relative to coordinate system  $\xi^*\eta^*$  so that it remains parallel to the  $\xi^*$ -axis. This stabilization can be obtained by free gyroscopes or by tracking stars (astronavigation) [2]. In this case, the newtonometer should be turned relative to the platform by the following angle

$$\chi = \frac{1}{R} \int_0^t a(t) dt^2 + \frac{v(0)}{R} t \quad (4.1)$$

for which its readings should be integrated twice.

With the precise observation of certain equations similar to the preceding equation, we have

$$s = R\chi \quad (4.2)$$

There are original devices which perform the double integration immediately without breaking it down into two successive operations (Boykov integrator) [3].

5. We will proceed to the consideration of the problem of determining the location of an object whose center moves randomly over the Earth's sphere. For solving this problem we will use a platform stabilized by gyroscopes, so that the angular velocities

$$\omega_x = \frac{M_1}{H}, \quad \omega_y = \frac{M_2}{H}, \quad \omega_z = \frac{M_3}{H} \quad (5.1)$$

around the x- and y-axes lying in the plane of the platform and the z-axis normal to it arose because three moments  $M_1$ ,  $M_2$  and  $M_3$  applied to the axes of the gyroscope housings.

Figure 4 shows a possible diagram for realizing this type of stabilization. The plane of the platform P, which is connected with the object by a three-dimensional cardan suspension (not shown in the figure), is continuously made to coincide with the plane perpendicular to the intrinsic axis of the gyroscope 1 by servo systems. The bearings of the axis of the outer ring of this

gyroscope's cardan suspension are arranged on the stabilized platform so that the axis itself lies in the plane of the platform. When the servo system operates perfectly, the axis of the inner ring, i.e., the axis of the gyroscope housing, also coincides with the plane of the platform. Let  $M_1$  and  $M_2$  be the moments applied to the axis of the outer ring of the suspension and the axis of the housing, respectively, designated by  $x$  and  $y$ . The moments  $M_1$  and  $M_2$  cause the precession of the gyroscope and, consequently, the rotation of the platform around the  $x$ - and  $y$ -axes at angular velocities  $\omega_x$  and  $\omega_y$ . The values of the latter are determined by the first two formulae in (5.1).

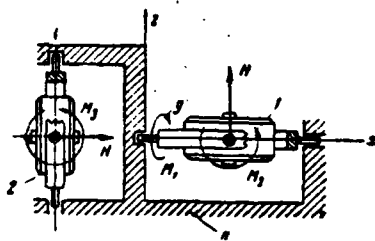


Fig. 4.

The bearings of the axis of the outer cardan ring of the gyroscope 2 (Fig. 4) are connected to the platform with the same intrinsic kinetic moment  $d$  as the first. The axis of this outer ring is perpendicular to the plane of the platform. A certain correcting

moment is applied to this axis so that the natural rotation axis of the gyroscope 1 is parallel to the plane of the platform. Furthermore, a special servo system continuously makes the above x-axis, which should be considered to be rigidly bound to the stabilized platform, coincide with the natural rotation axis of the gyroscope 2. The moment  $M_3$  applied to the axis of the housing of the second gyroscope causes the precession of the gyroscope and, consequently, the rotation of the platform parallel to the z-axis at angular velocity  $\omega_z$ . The latter is perpendicular to the plane of the platform, and together with the x- and y-axes, forms rectangular coordinate system xyz, which is rigidly bound to the platform. We will consider the origin of this system to be located in the center of the object. The values of  $M_3$  and  $\omega_z$  are related by the third formula of (5.1).

Two newtonometers whose axes of sensitivity are directed along the axes of coordinates x and y are located on the stabilized platform in the direct proximity of the origin of coordinate system xyz. Let the center of the moving object move randomly over the Earth's sphere, and the moments  $M_1$  and  $M_2$  be related to the readings of the corresponding newtonometers  $a_x$  and  $a_y$  by the formulae

$$M_1 = -K \int_0^t a_y dt - m_1, \quad M_2 = K \int_0^t a_x dt + m_2 \quad (5.2)$$

for which special integrating devices must be provided.

We will explain what parameters  $K$ ,  $m_1$  and  $m_2$ , and also moment  $M_3$  must be in order for the plane of the platform to remain horizontal.

The projections of the acceleration of the origin of coordinates relative to nonrotating sphere  $S$  on the  $x$ -,  $y$ - and  $z$ -axes is expressed by the formulae [4, 5]

$$\begin{aligned} w_x &= \frac{dv_x}{dt} + \omega_y v_z - \omega_z v_y \\ w_y &= \frac{dv_y}{dt} + \omega_z v_x - \omega_x v_z \\ w_z &= \frac{dv_z}{dt} + \omega_x v_y - \omega_y v_x \end{aligned} \quad (5.3)$$

where  $v_x$ ,  $v_y$  and  $v_z$  are the projections of the velocity of the origin of coordinates on the same axes relative to sphere  $S$ . In the case in question,  $v_z \equiv 0$ , and the force of gravity has projections equal to zero on the  $x$ - and  $y$ -axes. Therefore, the newtonometers directly measure accelerations  $w_x$  and  $w_y$ , and according to formulae (5.1) and (5.2), the following equations hold:

$$\begin{aligned} \omega_x &= -\frac{K}{H} \int_0^t \left( \frac{dv_y}{dt} + \omega_z v_z \right) dt - \frac{m_1}{H} \\ \omega_y &= \frac{K}{H} \int_0^t \left( \frac{dv_x}{dt} - \omega_z v_z \right) dt + \frac{m_2}{H} \end{aligned} \quad (5.4)$$

Since the platform must remain horizontal and the  $z$ -axis directed along the Earth's radius as the object moves, in equations

(5.4) we should [5] consider that

$$v_x = \omega_y R, \quad v_y = -\omega_x R \quad (5.5)$$

Using these formulae, we will eliminate the values  $\omega_x$  and  $\omega_y$  from equations (5.4). We will have

$$\begin{aligned} v_x &= \frac{KR}{H} \int_0^t \left( \frac{dv_x}{dt} - \omega_x v_y \right) dt + \frac{R}{H} m_2 \\ v_y &= \frac{KR}{H} \int_0^t \left( \frac{dv_y}{dt} + \omega_x v_x \right) dt + \frac{R}{H} m_1 \end{aligned} \quad (5.6)$$

Equations (5.6) must be satisfied during a random change in  $v_x$  and  $v_y$ , i.e., they must be identities. However, this is only possible with the following conditions:

$$\frac{KR}{H} = 1, \quad \omega_x \equiv 0, \quad m_1 = Kv_y(0), \quad m_2 = Kv_x(0) \quad (5.7)$$

The first condition coincides with equation (2.2); the second, according to the third formula of (5.1), leads to the requirement

$$M_3 = 0 \quad (5.8)$$

Finally, the last two conditions concern the agreement of the initial velocity of the center of the moving object (relative to sphere S), or to be more precise, its projection on the x- and y-axes, with parameters  $m_1$  and  $m_2$  of the integrating devices.

6. According to formulae (5.1), (5.2) and (5.8), as a result of the work of the integrators, the projections of the angular velocity of the stabilized platform  $\omega_x$  and  $\omega_y$  are now known time functions,  $\omega_z \equiv 0$ , and the z-axis is directed along the Earth's radius. Theoretically, this makes it possible to continuously determine the location of the object on the Earth's sphere and its course.

Actually, we will consider (Fig. 5) the so-called geographic triangle  $\xi\eta\zeta$ , whose apex coincides with the origin of coordinate system xyz (i.e., with the center of the object); side  $\xi$  is directed towards the east, side  $\eta$  - towards the north and, finally, side  $\zeta$  - from the Earth's radius upward. The projections of the angular velocity  $u$  of this triangle relative to nonrotating sphere  $S$  on sides  $\xi$ ,  $\eta$  and  $\zeta$  are expressed [6] by the formulae

$$u_\xi = -\frac{V_N}{R}, \quad u_\eta = \frac{V_E}{R} + U \cos \varphi, \quad u_\zeta = \frac{V_E}{R} \operatorname{tg} \varphi + U \sin \varphi \quad (6.1)$$

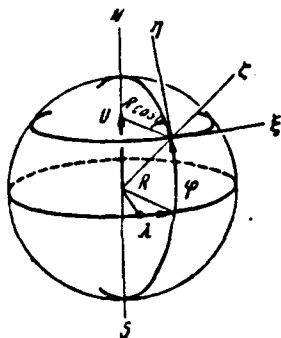


Fig. 5.

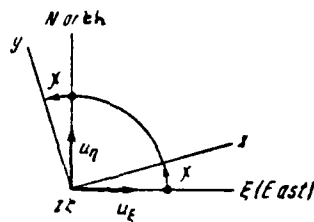


Fig. 6.

Here  $U$  is the angular velocity of the Earth,  $\phi$  is the current latitude of the location, and  $V_E$  and  $V_N$  are the eastern and northern components of the velocity of the center of the object relative to the Earth, respectively. Obviously,

$$V_E = R \cos \phi \frac{d\lambda}{dt}, \quad V_N = R \frac{d\phi}{dt} \quad (6.2)$$

where  $\lambda$  is the current longitude.

In this case, side  $\zeta$  coincides with the  $z$ -axis. Therefore, the projections  $\omega_x$ ,  $\omega_y$  and  $\omega_z$  are related to the values  $\omega_x$ ,  $\omega_y$  and  $\omega_z$  by the relationships



$$\begin{aligned}\omega_x &= u_\xi \cos \chi + u_\eta \sin \chi \\ \omega_y &= -u_\xi \sin \chi + u_\eta \cos \chi, \quad \omega_z = u_\xi + \frac{d\chi}{dt}\end{aligned}\quad (6.3)$$

where  $\chi$  is the angle between the  $\xi$ - and  $x$ -axes (Fig. 6).

Considering formulae (6.1) and (6.2) and the fact that  $\omega_z \equiv 0$ , and that  $\omega_x = \omega_x(t)$  and  $\omega_y = \omega_y(t)$  are unknown time functions, according to relationships (6.3) we will arrive at a system of three differential equations:

$$\begin{aligned}-\frac{d\varphi}{dt} \cos \chi + \left(U + \frac{d\lambda}{dt}\right) \cos \varphi \sin \chi &= \omega_x(t) \\ \frac{d\varphi}{dt} \sin \chi + \left(U + \frac{d\lambda}{dt}\right) \cos \varphi \cos \chi &= \omega_y(t) \\ \left(U + \frac{d\lambda}{dt}\right) \sin \varphi + \frac{d\chi}{dt} &= 0\end{aligned}\quad (6.4)$$

for the three unknown functions  $\varphi(t)$ ,  $\lambda(t)$  and  $\chi(t)$ .

With known initial conditions  $\varphi(0)$ ,  $\lambda(0)$  and  $\chi(0)$ , i.e., data on the location of the object and its orientation at the initial point in time, equations (6.4) can be integrated on a special computer. Here it is helpful to solve them first for the derivatives, i.e., to represent them in the form

$$\begin{aligned}
 \frac{d\varphi}{dt} &= -\omega_x(t) \cos \chi + \omega_y(t) \sin \chi \\
 \frac{d\lambda}{dt} &= -U + \frac{\omega_x(t) \sin \chi + \omega_y(t) \cos \chi}{\cos \varphi} \\
 \frac{d\chi}{dt} &= -[\omega_x(t) \sin \chi + \omega_y(t) \cos \chi] \operatorname{tg} \varphi
 \end{aligned}
 \quad (6.5)$$

Having determined functions  $\varphi(t)$  and  $\chi(t)$ , we can also find the course of the object, i.e., the angle  $\kappa$  which its velocity vector forms relative to the Earth's surface with the meridian of the location (Fig. 7). According to formulae (6.2) and (6.5), the course  $\kappa$  is determined by the equation

$$\operatorname{tg} \kappa = \frac{U \cos \varphi - \omega_x(t) \sin \chi - \omega_y(t) \cos \chi}{\omega_x(t) \cos \chi - \omega_y(t) \sin \chi} \quad (6.6)$$

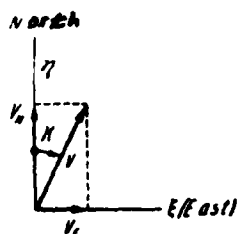


Fig. 7.

7. Now we will proceed to studying the small movements of the

stabilized platform with the assumption that at the initial instant, its plane was not in a precisely horizontal position and conditions (5.7) of the selection of the value of parameters  $m_1$  and  $m_2$  have been satisfied with a small error. Furthermore, we will consider that the first two equations of (5.7) have been realized with complete precision.

We will introduce the Darboux triangle  $x^0y^0z^0$ , whose sides  $x^0$  and  $y^0$  are tangent to the Earth's sphere [5, 7] and, consequently, also nonrotating sphere  $S$ . Side  $x^0$  is directed along the velocity vector  $v$  of the apex of the triangle relative to sphere  $S$ . We will call triangle  $x^0y^0z^0$  a natural Darboux triangle. We will place its apex in the center of the moving object, i.e., at the origin of coordinate system  $xyz$ , which is rigidly bound to the stabilized platform. The projections of the angular velocity  $\omega^0$  of the natural Darboux triangle on sides  $x^0$ ,  $y^0$  and  $z^0$  are represented by the formulae

$$\omega_{x^0} = 0, \quad \omega_{y^0} = \frac{v}{R}, \quad \omega_{z^0} = \tilde{\omega} \quad (7.1)$$

At a given velocity  $v = v(t)$ , functions  $\tilde{\omega} = \tilde{\omega}(t)$  determine [7] the geodesic curvature of the trajectory of the apex of triangle  $x^0y^0z^0$  on sphere  $S$ .

In turn, the projections of the acceleration of the apex of the triangle during its movement relative to nonrotating sphere S onto sides  $x^0$ ,  $y^0$  and  $z^0$  are expressed by the formulae

$$w_{x^0} = \frac{dv}{dt}, \quad w_{y^0} = \tilde{\omega}v, \quad w_{z^0} = -\frac{v^3}{R} \quad (7.2)$$

Henceforth, we will use the table of cosines of the angles between the axes of coordinate system  $xyz$  and the sides of the triangle  $x^0y^0z^0$ . It is:

	$x^0$	$y^0$	$z^0$
$x$	$\cos \gamma \cos \beta - \sin \gamma \sin \alpha \sin \beta$	$\sin \gamma \cos \beta + \cos \gamma \sin \alpha \sin \beta$	$-\cos \alpha \sin \beta$
$y$	$-\sin \gamma \cos \alpha$	$\cos \gamma \cos \alpha$	$\sin \alpha$
$z$	$\cos \gamma \sin \beta + \sin \gamma \sin \alpha \cos \beta$	$\sin \gamma \sin \beta - \cos \gamma \sin \alpha \cos \beta$	$\cos \alpha \cos \beta$

(7.3)

Here the angles  $\alpha$ ,  $\beta$ ,  $\gamma$  describe the orientation of coordinate system  $xyz$  relative to triangle  $x^0y^0z^0$  (Fig. 8). The angle  $\gamma$  is the angle of rotation of the auxiliary coordinate system  $x'y'z'$  relative to the triangle. The  $z'$ -axis of this system coincides with side  $z^0$ . The coordinate system is rotated counterclockwise (viewed from the positive part of the  $z'$ -axis) until the  $x'$ -axis coincides with plane  $zx$ . Similarly, angle  $\alpha$  is determined by the relative position of coordinate system  $x'y'z'$  and another auxiliary system  $x''y''z''$ , the  $x'$ - and  $x''$ -axes of which coincide. The  $z''$ -axis of the latter coordinate system also coincides with plane  $zx$ , as a result of which its axis  $y''$

turns out to be directed along the  $y$ -axis.

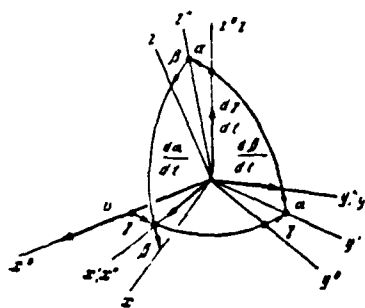


Fig. 8.

When  $\alpha > 0$ , coordinate system  $x''y''z''$  is turned counterclockwise relative to system  $x'y'z'$ , if we observe its rotation from the positive part of the  $x'$ -axis (or, analogously, the  $x''$ -axis).

Finally, angle  $\beta$  is the angle between  $x$ - and  $x'$ -axes of coordinate systems  $xyz$  and  $x''y''z''$ . The sign of angle  $\beta$  is determined analogously to the signs of angles  $\gamma$  and  $\alpha$ .

The angular velocity  $\omega$  of coordinate system  $xyz$  relative to nonrotating sphere  $S$  is the geometric sum of the angular velocity  $\omega^0$  of the natural Darboux triangle relative to the same sphere and the three relative angular velocities  $d\gamma/dt$ ,  $d\alpha/dt$  and  $d\beta/dt$ . The latter are the angular velocities of coordinate system  $x'y'z'$  relative to

triangle  $x^0y^0z^0$ , of system  $x''y''z''$  relative to  $x'y'z'$ , and, finally, of coordinate system  $xyz$ , i.e., the stabilized platform, relative to system  $x''y''z''$ .

The relative angular velocity  $d\gamma/dt$  is directed along side  $z^0$ , the angular velocity  $d\alpha/dt$  - along the  $x'$ -axis, and angular velocity  $d\beta/dt$  - along the  $y$ -axis. The  $x'$ -axis coincides with side  $x^0$  with an angle  $\gamma = 0$  which, according to Table (7.3), makes it possible to find the cosines of the angles formed by the direction of the relative angular velocity  $d\alpha/dt$  to the axis of coordinate system  $xyz$  (Fig. 8). Considering all of the above, we obtain the following expressions for the projections  $\omega_x$ ,  $\omega_y$  and  $\omega_z$  of the angular velocity of the stabilized platform onto the axes of coordinate system  $xyz$ , which is rigidly bound to it:

$$\begin{aligned}\omega_x &= \frac{v}{R} (\sin \gamma \cos \beta + \cos \gamma \sin \alpha \sin \beta) + \left( \tilde{\omega} + \frac{d\gamma}{dt} \right) (-\cos \alpha \sin \beta) + \frac{d\alpha}{dt} \cos \beta \\ \omega_y &= \frac{v}{R} \cos \gamma \cos \alpha + \left( \tilde{\omega} + \frac{d\gamma}{dt} \right) \sin \alpha + \frac{d\beta}{dt} \\ \omega_z &= \frac{v}{R} (\sin \gamma \sin \beta - \cos \gamma \sin \alpha \cos \beta) + \left( \tilde{\omega} + \frac{d\gamma}{dt} \right) \cos \alpha \cos \beta + \frac{d\alpha}{dt} \sin \beta\end{aligned}\quad (7.4)$$

Using table (7.3) and formulae (7.2), we then arrive at the expressions for the projections  $w_x$ ,  $w_y$  and  $w_z$  of the acceleration of the origin of coordinate system  $xyz$ , namely

$$\begin{aligned}
w_x &= \frac{dv}{dt} (\cos \gamma \cos \beta - \sin \gamma \sin \alpha \sin \beta) + \tilde{\omega} v (\sin \gamma \cos \beta + \cos \gamma \sin \alpha \sin \beta) - \\
&\quad - \frac{v^2}{R} (-\cos \alpha \sin \beta) \\
w_y &= \frac{dv}{dt} (-\sin \gamma \cos \alpha) + \tilde{\omega} v \cos \gamma \cos \alpha - \frac{v^2}{R} \sin \alpha \\
w_z &= \frac{dv}{dt} (\cos \gamma \sin \beta + \sin \gamma \sin \alpha \cos \beta) + \tilde{\omega} v (\sin \gamma \sin \beta - \cos \gamma \sin \alpha \cos \beta) - \\
&\quad - \frac{v^2}{R} \cos \alpha \cos \beta
\end{aligned} \tag{7.5}$$

In this case, unlike §6, the projections of the force of gravity on the x- and y-axes are already nonzero. Therefore, the newtonometers located on the x- and y-axes will measure the sum of accelerations  $w_x$  and  $w_y$  corresponding to the projections (with the opposite signs) of the acceleration of the force of gravity  $j$  onto these same axes. This acceleration is directed along side  $z^0$  toward the center of the Earth. Now, considering formulae (7.5) and (7.3), we will find that the readings  $a_x$  and  $a_y$  of the corresponding newtonometers should be expressed by the formulae

$$\begin{aligned}
a_x &= \frac{dv}{dt} (\cos \gamma \cos \beta - \sin \gamma \sin \alpha \sin \beta) + \tilde{\omega} v (\sin \gamma \cos \beta + \cos \gamma \sin \alpha \sin \beta) + \\
&\quad + \left(j - \frac{v^2}{R}\right) (-\cos \alpha \sin \beta) \\
a_y &= \frac{dv}{dt} (-\sin \gamma \cos \alpha) + \tilde{\omega} v \cos \gamma \cos \alpha + \left(j - \frac{v^2}{R}\right) \sin \alpha
\end{aligned} \tag{7.6}$$

Now we will form the moments  $M_1$  and  $M_2$  according to formulae (5.2), while  $M_3$ , like before, will be considered to be equal to zero. According to relationships (5.1), we will have

$$\omega_x = -\frac{K}{H} \int_0^t a_y dt - \frac{m_1}{H}, \quad \omega_y = \frac{K}{H} \int_0^t a_x dt + \frac{m_2}{H}, \quad \omega_z = 0 \quad (7.7)$$

As follows from formulae (7.4) and (7.6), these equations can be considered to be the system of equations for determining the time functions  $\alpha(t)$ ,  $\beta(t)$  and  $\gamma(t)$  with the given functions  $v(t)$  and  $\tilde{\omega}(t)$ .

Differentiating the first two equations of (7.7) with respect to time, we will arrive at a system of differential equations for the same unknown functions:

$$\frac{d\omega_x}{dt} + \frac{K}{H} a_y = 0, \quad \frac{d\omega_y}{dt} - \frac{K}{H} a_x = 0, \quad \omega_z = 0 \quad (7.8)$$

Considering angles  $\alpha$  and  $\beta$  in formulae (7.4) and (7.6) to be small, and disregarding the second-order terms relative to these variables, we will reduce the expressions for  $\omega_x, \omega_y, \omega_z, a_x$  and  $a_y$  to

$$\begin{aligned} \omega_x &= \frac{v}{R} \sin \gamma - \left( \tilde{\omega} + \frac{d\gamma}{dt} \right) \beta + \frac{da}{dt} \\ \omega_y &= \frac{v}{R} \cos \gamma + \left( \tilde{\omega} + \frac{d\gamma}{dt} \right) \alpha + \frac{d\beta}{dt} \\ \omega_z &= \frac{v}{R} (\beta \sin \gamma - \alpha \cos \gamma) + \tilde{\omega} + \frac{d\gamma}{dt} \\ a_x &= \frac{dv}{dt} \cos \gamma + \tilde{\omega} v \sin \gamma - \left( j - \frac{v^2}{R} \right) \beta \\ a_y &= -\frac{dv}{dt} \sin \gamma + \tilde{\omega} v \cos \gamma + \left( j - \frac{v^2}{R} \right) \alpha \end{aligned} \quad (7.9)$$

Since  $\omega_z = 0$ , according to the third formula of (7.9),



$$\ddot{\omega} + \frac{d\gamma}{dt} = \frac{v}{R} (\alpha \cos \gamma - \beta \sin \gamma) \quad (7.10)$$

Whence it follows that the terms

$$\left(\ddot{\omega} + \frac{d\gamma}{dt}\right)\alpha, \quad \left(\ddot{\omega} + \frac{d\gamma}{dt}\right)\beta \quad (7.11)$$

contained in the first two formulae of (7.9) are of the second order of smallness relative to variables  $\alpha$  and  $\beta$  and, consequently, they can be dropped. Thus, with precision down to the second order of smallness, we will have

$$\omega_x = \frac{v}{R} \sin \gamma + \frac{d\alpha}{dt}, \quad \omega_y = \frac{v}{R} \cos \gamma + \frac{d\beta}{dt} \quad (7.12)$$

Now, if we use relationship (7.10) to eliminate function  $\ddot{\omega}(t)$  from the fourth and fifth formulae of (7.9), after simple transformations we will obtain the following expressions for the values of  $a_x$  and  $a_y$ :

$$\begin{aligned} a_x &= \frac{d}{dt} (v \cos \gamma) + \frac{v^2}{R} (\alpha \cos \gamma - \beta \sin \gamma) \sin \gamma - \left(j - \frac{v^2}{R}\right) \beta \\ a_y &= -\frac{d}{dt} (v \sin \gamma) + \frac{v^2}{R} (\alpha \cos \gamma - \beta \sin \gamma) \cos \gamma + \left(j - \frac{v^2}{R}\right) \alpha \end{aligned} \quad (7.13)$$

Finally, substituting the values of  $\omega_x, \omega_y, a_x$  and  $a_y$  in the first two equations of (7.8) according to formulae (7.12) and (7.13) and considering equation (2.2), after obvious simplifications, we will

arrive at two second-order differential equations:

$$\begin{aligned}\frac{d^2\alpha}{dt^2} + \frac{1}{R}\alpha &= \frac{v^2}{R^2}(\alpha \sin \gamma + \beta \cos \gamma) \sin \gamma \\ \frac{d^2\beta}{dt^2} + \frac{1}{R}\beta &= \frac{v^2}{R^2}(\alpha \sin \gamma + \beta \cos \gamma) \cos \gamma\end{aligned}\quad (7.14)$$

Together with relationship (7.10), they form a system of differential equations for the functions  $\alpha(t)$ ,  $\beta(t)$  and  $\gamma(t)$ .

Considering the smallness of the values of  $\alpha$  and  $\beta$ , angle  $\gamma$  in equations (7.14) can be replaced by the integral

$$\tilde{\gamma} = - \int_0^t \tilde{\omega} dt + \gamma_0 \quad (7.15)$$

where  $\gamma_0$  is the value of the angle  $\gamma$  between the x-axis and side  $x^0$  (i.e., the velocity vector of the center of the object) at the initial point in time. As it is easy to confirm from equations (7.10) and (7.14), the errors which arise during this substitution are of the second order of smallness.

Thus, the study of small oscillations of a stabilized platform is reduced to the integration of two linear differential equations with variable coefficients depending on time<sup>1</sup>.

Footnote: 'It is curious to note the complete identity of equations (7.14) of small oscillations of the stabilized platform in question with the equations of small oscillations of a certain physical pendulum with a reference point moving over sphere S. The equilibrium conditions of this pendulum relative to the natural Darboux triangle are given in report [5]. End footnote

8. Setting aside the integration of the differential equations (7.14) of small oscillations of the platform during random movement of the center of the object over the Earth's sphere, we will limit ourselves to the case  $\tilde{\omega} = 0$  and  $v = \text{const}$ , which corresponds to movement at a constant velocity over an arc of the great circle of nonrotating sphere S. In this case, movement relative to the Earth will occur with a variable relative velocity over a complex trajectory. Setting  $\tilde{\omega} = 0$  in formula (7.15), we conclude that the angle  $\gamma$  in system of differential equations (7.14) should be considered to be constant. As a result, system (7.14) is broken down into two independent equations:

$$\begin{aligned} \frac{d^2}{dt^2} (\alpha \sin \gamma + \beta \cos \gamma) + \left( \frac{1}{R} - \frac{r^2}{R^3} \right) (\alpha \sin \gamma + \beta \cos \gamma) &= 0 \\ \frac{d^2}{dt^2} (\alpha \cos \gamma - \beta \sin \gamma) + \frac{1}{R} (\alpha \cos \gamma - \beta \sin \gamma) &= 0 \end{aligned} \quad (8.1)$$

The first of them corresponds to the angular oscillations of the

platform around the side  $y^0$  of the natural Darboux triangle, and the second - around side  $x^0$ , respectively. As we already mentioned above, side  $x^0$  is directed along velocity vector  $v$ . The frequencies of these oscillations are close together, as long as the value of the velocity  $v$  is not too great (e.g., a value less than the velocity of points on the Earth's equator during its diurnal rotation). This corresponds to a period of time approximately equal to 84 min. (i.e., the Schuler period).

9. In the presence of small oscillations of a stabilized platform, i.e., when angles  $\alpha$  and  $\beta$  are nonzero, relationships (5.5) will be realized with a certain error. Furthermore, the initial equations (5.6) for this method of autonomous determination of location cannot be turned into precise identities because of the presence of projections of the acceleration of gravity  $j$  in the newtonometer readings  $a_x$  and  $a_y$ . One would expect the errors in determining the latitude and longitude of the location of the moving object and its course, which occur because of the above circumstances, to fluctuate. However, additional research is necessary for the precise determination of the nature of the change in these errors through time.

10. Above it was assumed that the center of the object moves over the sphere S, on the basis of which it was assumed that  $v_z = 0$  in formulae (5.3). Now we will show how to eliminate this restrictive condition.

Suppose that a platform stabilized by gyroscopes moves so that its plane remains perpendicular to the Earth's radius. In this case, it is necessary to satisfy the same equations (5.5), where  $v_x$  and  $v_y$ , like before, are the projections of the velocity  $v$  of the center of the cardan suspension of the platform onto the x- and y-axes relative to the sphere S, and  $\omega_x$  and  $\omega_y$  are the projections of its angular velocity  $\omega$  onto the same axes. Thus, we should have

$$\omega_x = -\frac{v_y}{R}, \quad \omega_y = \frac{v_x}{R} \quad (10.1)$$

Unlike equations (5.5), here  $R = R(t)$  is the variable value of the distance between the center of the cardan suspension of the platform and the center of sphere S.

The angular velocities  $\omega_x$  and  $\omega_y$  are created by applying moments  $M_1$  and  $M_2$  to the platform according to the first two formulae of (5.1). Consequently, moments  $M_1$  and  $M_2$  should be formed according to the equations

$$M_2 = -\frac{H}{R} v_y, \quad M_1 = \frac{H}{R} v_z \quad (10.2)$$

We will place two newtonometers, whose axes of sensitivity are directed along the x- and y-axes, which lie in the plane of the platform, in direct proximity to the center of the cardan suspension. Their readings  $a_x(t)$  and  $a_y(t)$  will not contain the projections of the acceleration of gravity  $j$  onto the x- and y-axes, since by assumption, as it moves the stabilized platform remains perpendicular to the straight line connecting the center of the cardan suspension and the center of sphere S. Based on this, we can set

$$a_x(t) = w_x = \frac{dv_x}{dt} + \omega_y v_z - \omega_z v_y, \quad a_y(t) = w_y = \frac{dv_y}{dt} + \omega_z v_x - \omega_x v_z \quad (10.3)$$

Here, unlike the case in §6,  $v_z$  is already nonzero and is expressed as

$$v_z = \frac{dR}{dt} \quad (10.4)$$

Like before, if we consider  $M_3 = 0$ , then according to the third formula of (5.1), we again obtain  $\omega_z \equiv 0$ , i.e., the platform will not have its angular velocity component  $\omega$  along the z-axis directed along the radius of sphere S. Considering this, as well as formulae (10.1)

and (10.4), according to equations (10.3), we arrive at the relationships

$$a_x(t) = \frac{dv_x}{dt} + \frac{v_x}{R} \frac{dR}{dt}, \quad a_y(t) = \frac{dv_y}{dt} + \frac{v_y}{R} \frac{dR}{dt} \quad (10.5)$$

These relationships can be considered to be differential equations which make it possible to plot functions  $v_x(t)$  and  $v_y(t)$  with known functions  $a_x(t)$  and  $a_y(t)$ , as well as  $R = R(t)$ . The values of the former functions are necessary for forming moments  $M_1$  and  $M_2$ , which control the orientation of the platform.

The solution of equations (10.5) is reduced to the quadratic equations

$$v_x = \frac{1}{R} \left[ \int_0^t R a_x(t) dt + R(0) v_x(0) \right], \quad v_y = \frac{1}{R} \left[ \int_0^t R a_y(t) dt + R(0) v_y(0) \right] \quad (10.6)$$

Thus, according to formulae (10.2), moments  $M_1$  and  $M_2$  should be formed from the newtonmeter readings  $a_x(t)$  and  $a_y(t)$  as follows:

$$\begin{aligned} M_2 &= -\frac{H}{R^2} \left[ \int_0^t R(t) a_y(t) dt + R(0) v_y(0) \right] \\ M_1 &= \frac{H}{R^2} \left[ \int_0^t R(t) a_x(t) dt + R(0) v_x(0) \right] \end{aligned} \quad (10.7)$$

For this purpose, devices which multiply and divide the current values must be provided, as well as integrators.

The variable value of  $R = R(t)$  in equations (10.7) is considered to be known in advance. However, the presence of a third newtonometer whose axis of sensitivity is parallel to the z-axis (the Earth's radius) theoretically makes it possible to determine this function independent of any other auxiliary devices. Actually, the reading of this newtonometer is determined by the formula

$$a_z = w_z - j = \frac{dv_z}{dt} + \omega_x v_y - \omega_y v_x - j \quad (10.8)$$

Here

$$j = j_0 \frac{R_0^2}{R^3} \quad (10.9)$$

the acceleration of gravity, which decreases with the increase in the distance from the Earth's center,  $j_0$  is its value on the Earth's surface, whose radius is designated as  $R_0$ .

Using formulae (10.1), (10.4) and (10.9), we obtain the following for function  $R(t)$

$$\frac{d^2 R}{dt^2} - \frac{v_x^2 + v_y^2}{R} - \frac{j_0 R_0^2}{R^3} = a_z(t) \quad (10.10)$$

The device which integrates this differential equation must be connected to one system with integrators which reproduce moments  $M_z$



and  $M_1$ . According to formulae (10.2), these moments differ from functions  $v_y(t)$  and  $v_x(t)$  only in the constant factor. Actually, functions  $v_x(t)$  and  $v_y(t)$  are in equation (10.10); in turn, function  $R(t)$  is used in formulae (10.0).

Problems of the stability of this computational system require special examination. Without discussing the study of small oscillations of the stabilizing platform, either, we will point out that the subsequent solution of the problem of the location of a moving object is reduced to the integration of the same system of differential equations (6.5), where  $\omega_x(t)$  and  $\omega_y(t)$  are considered to be known functions of time on the basis of formulae (10.1), (10.2) and (10.6).

Received 20 May 1957

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